



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

## MECHANICS.

**304. Proposed by B. F. FINKEL, Drury College.**

A spherical shell, inner radius  $r$  and outer radius  $R$ , has within it a perfectly smooth solid sphere of the same material and with radius  $r_1 < r$ . If the inner surface of the spherical shell is also perfectly smooth, determine the motion, after the time  $t$ , of the shell and sphere down a rough inclined plane, inclination  $\alpha$ .

## II. SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

Let the radii of gyration of the shell and sphere,  $k'$ ,  $k''$ , be given by

$$k'^2 = \frac{2}{5} \frac{R^5 - r^5}{R^3 - r^3},$$

$$k''^2 = \frac{2}{5} r_1^2;$$

$C_0T_0$  the radius of the shell to the tangent point on the inclined plane initially;  $E_0$  the common point initially of the inner surface of the shell and sphere;  $CT_0$  the position of  $C_0T_0$  after any time from the beginning of motion of the system;  $D$  the center of the sphere at the same time  $t$ ;  $CT$  the radius of the shell to the tangent point  $T$  of shell and inclined plane at the same instant;  $\varphi$  = angle  $TCT_0$ ;  $DE$  = the radius of the sphere to the tangent point of the inner surface of the shell and the sphere;  $\theta$  = the angle  $DE$  makes with the vertical through  $D$ ;

$$s = TT_0 = CC_0 = R\varphi;$$

$CH$  = a perpendicular from  $C$  upon  $C_0E_0$  cutting the latter at  $H$ ;

$$r - r_1 = \dot{r};$$

$C_0$  the coördinate origin;  $C_0C$  the  $x$ -axis;  $C_0T_0$  the  $y$ -axis;  $x$ ,  $y$  the coördinates of  $D$ ; then  $\angle C_0CH = \alpha$ ;  $\angle C_0CD = \pi/2 - \theta + \alpha$ ; and then,  $F$  being the foot of the perpendicular from  $D$  upon  $CC_0$ ,

$$x = s - CF = R\varphi - r' \cos DCF = R\varphi - r' \sin(\theta - \alpha);$$

$$y = DF = r' \sin DCF = r' \cos(\theta - \alpha).$$

The dynamic conditions for the motion of the sphere can be most clearly indicated by noticing that the initial point  $E_0$  remains in contact with the inner surface of the shell, while the sphere has an angular velocity  $\dot{\theta}$ ,  $\dot{\varphi}$  being that of the shell.

Let  $M$ ,  $m$ , be the masses of the shell and of the sphere;  $T$ ,  $V$ , the kinetic energy, and potential energy; then the kinetic potential equation for the system is

$$\begin{aligned} T &= \frac{1}{2}M(k'^2\dot{\varphi}^2 + \dot{s}^2) + \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + k''^2\dot{\theta}^2) \\ &= Mgs \sin \alpha + mg(r' \cos \theta + s \sin \alpha) + C = V. \end{aligned}$$

But

$$\dot{x} = R\dot{\varphi} - r' \cos(\theta - \alpha)\dot{\theta},$$

$$\dot{y} = -r' \sin(\theta - \alpha)\dot{\theta},$$

and

$$\dot{s} = R\dot{\varphi},$$

whence, on substitution and reduction,

$$\begin{aligned} T &= \frac{1}{2}\{M(k'^2 + R^2) + mR^2\}\dot{\varphi}^2 + \frac{1}{2}m\{-2Rr' \cos(\theta - \alpha)\dot{\theta}\dot{\varphi} + (r'^2 + k''^2)\dot{\theta}^2\} \\ &= g\{(M + m)R\dot{\varphi} \sin \alpha + mr' \cos \theta\} + C = V. \end{aligned}$$

The Lagrangian equations

$$\frac{d}{dt} \frac{dT}{d\dot{\varphi}} - \frac{dT}{d\varphi} = \frac{dV}{d\varphi},$$

$$\frac{d}{dt} \frac{dT}{d\dot{\theta}} - \frac{dT}{d\theta} = \frac{dV}{d\theta},$$

applied to the last result, give after simplifications,

$$\{(M+m)R^2 + Mk'^2\}\ddot{\phi} - mRr'\cos(\theta-\alpha)\ddot{\theta} + mRr'\sin(\theta-\alpha)\dot{\theta}^2 = g(M+m)R\sin\alpha,$$

$$Rr'\cos(\theta-\alpha)\dot{\phi} - (r'^2 + k'^2)\ddot{\theta} = gr'\sin\theta.$$

Eliminating  $\ddot{\phi}$ ,

$$\begin{aligned} [(r'^2 + k'^2)\{(M+m)R^2 + Mk'^2\} + mR^2r'^2\cos^2(\theta-\alpha)]\ddot{\theta} - mR^2r'^2\sin(\theta-\alpha)\cos(\theta-\alpha)\dot{\theta}^2 \\ = -gr'[(\{(M+m)R^2 + Mk'^2\}\sin\theta + (M+m)R^2\sin\alpha\cos(\theta-\alpha)]. \end{aligned}$$

Multiplying by  $2\dot{\theta}$  and integrating

$$\begin{aligned} [(r'^2 + k'^2)\{(M+m)R^2 + Mk'^2\} + mr'^2R^2\cos^2(\theta-\alpha)]\dot{\theta}^2 \\ = gr'[2\{(M+m)R^2 + Mk'^2\}\cos\theta - 2(M+m)R^2\sin\alpha\sin(\theta-\alpha)] + C', \end{aligned}$$

which is of the same general form as (7), p. 351, this MONTHLY for November, 1916.

### NUMBER THEORY.

#### 235. Proposed by W. D. CAIRNS, Oberlin College.

Prove that  $n = 1$  is the only positive integer for which  $n^4 + 4$  is a prime.

SOLUTION BY WM. E. PATTEN, Government Institute of Technology, Shanghai, China.

$$n^4 + 4 = (n^4 + 4n^2 + 4) - 4n^2 = (n^2 + 2)^2 - (2n)^2 = (n^2 + 2n + 2)(n^2 - 2n + 2).$$

Therefore,  $n^4 + 4$  is a prime, if at all, only for those values of  $n$  which make either  $n^2 + 2n + 2 = 1$ , or  $n^2 - 2n + 2 = 1$ , since each of the factors of  $n^4 + 4$  given above is integral in value when  $n$  is integral, and both are positive when  $n$  is positive.

(1) When  $n^2 + 2n + 2 = 1$ , then  $n = -1$ .

(2) When  $n^2 - 2n + 2 = 1$ , then  $n = +1$ . When  $n = +1$ , then  $n^4 + 4 = 5$ , a prime. Therefore,  $n^4 + 4$  is a prime for  $n = 1$ , and for no other positive integral values of  $n$ .

Also solved by ELMER SCHUYLER, FRANK IRWIN, HORACE OLSON, ELIJAH SWIFT, H. H. CLARK, ELIZABETH B. DAVIS, NORMAN ANNING, L. G. WELD, and the PROPOSER.

#### 236. Proposed by V. M. SPUNAR, Chicago, Illinois.

Find integral values of  $x, y, z$ , such that

$$xy + z = \square, \quad yz + x = \square, \quad \text{and} \quad xz + y = \square.$$

SOLUTION BY ARTEMAS MARTIN, LL.D., Washington, D. C.

Assume  $x = n^2$ ,  $y = (n+1)^2$ ; then the given equation becomes

$$n^2(n+1)^2 + z = \square, \quad (n+1)^2z + n^2 = \square, \quad n^2z + (n+1)^2 = \square.$$

Put

$$n^2(n+1)^2 + z = a^2.$$

Assume  $a = n^2 + n + b$ , and the last equation becomes

$$n^2(n+1)^2 + z = a^2 = (n^2 + n + b)^2,$$

from which we immediately find

$$z = b(2n^2 + 2n + b).$$

Substituting in

$$(n+1)^2z + n^2 = \square,$$

we have

$$b(n+1)^2(2n^2 + 2n + b) + n^2 = \square = c^2,$$